DESIGN OF FRP LAMINATES USING EVOLUTIONARY ALGORITHMS

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1 INTRODUCTION

Fibre reinforced composites (FRP) are increasingly becoming important for civil engineering applications. Despite their relatively recent entry into civil engineering, strengthening methods based on FRP composites are gaining wide acceptance and an increasing number of structures all over the world have been strengthened using FRP, overcoming several of the disadvantages associated to steel based reinforcement techniques; they have a higher strength-to-weight ratio than steel, lower maintenance costs, they are easily delivered, handled and installed and are versatile for adapting to any structural shape.

As a result of the growing interest, some code proposals or recommendations [1-4] have been published in different countries or continents for the design of FRP strengthening systems for reinforced concrete (RC) structures. Since these guidelines were first published, continuous research has allowed to better understanding the behaviour of FRP strengthening devices and still some topics need further investigation. These proposals cover the flexural strengthening of beams and slabs, shear strengthening of beams and columns and flexural and compressive strengthening of columns. This work is focused on flexural and shear strengthening.

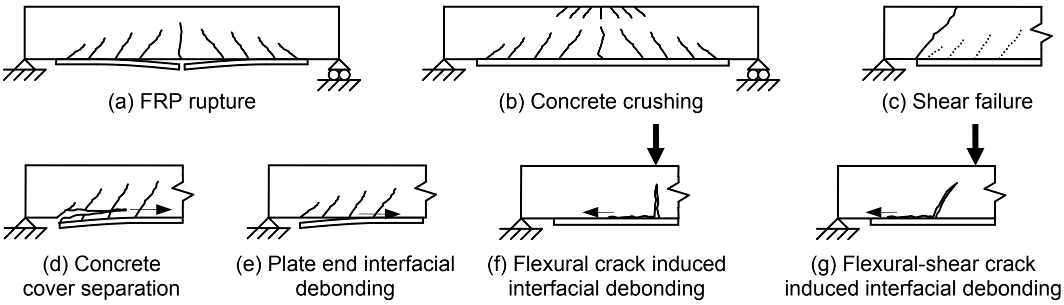

Fig. 1 Flexural strengthening: failure modes

For the flexural strengthening of RC beams with externally bonded FRP soffit plates, different failure modes are reported in literature [5, 6]. In the case of flexural strengthening, failure can happen when exceeding the tensile strength of the FRP soffit plate or the compressive strength of the concrete, but most frequently, plate interfacial debonding or concrete cover separation are reported [see Figure 1]. Also in the case of shear strengthening, interfacial debonding of the FRP sheets or strips is more frequently reported than rupture of the composite material. The effect of the flexural strengthening system is the stiffening of the RC beam, and as a result, the flexural behaviour is less ductile than in a non-strengthened member. Besides, interfacial debonding failure occurs in a brittle manner, which is unacceptable from the point of view of structural safety. Consequently, many research works have focused on this important issue through both experimental and theoretical investigations and the existing design recommendations warn against this kind of brittle failure mode. Interfacial debonding is highly dependent on the amount of external FRP reinforcement; for example, some tests results and experiences show that thicker soffit plates, which would be supposed to increase the flexural capacity, are however more prone to delamination [3]. Besides, FRP sheets or strips for shear strengthening, if properly installed, act as a kind of mechanical anchorage of the FRP soffit plate for flexural, which is therefore less prone to fail because of concrete cover separation [5]. Also, the shear capacity of a strengthened member should be higher than the flexural capacity, since the shear failure is always more brittle. The interaction between all the parameters involved in the design justifies the application of an optimization philosophy.

This work is aimed at obtaining an optimum design for the flexural and shear FRP strengthening system of simply supported RC beams, subjected to the limitations and recommendations specified in [1]. For that purpose, evolutionary optimization has been used.

2 OBJECTIVE FUNCTION AND DESIGN VARIABLES

The formulation of the present optimization problem is minimizing the cost of the repairing or retrofitting (which is the objective function), subject to a series of design constraints specified in [1]. The cost depends not only on the volume of FRP material used in the FRP reinforcement, but on the surface preparation work needed and the amount of adhesive required, as well. The mathematical form of the objective function is the following:

$$\text{Minimize} \quad C = w_1 \cdot V_{f,flex} + w_2 \cdot S_{f,flex} + w_3 \cdot V_{f,shear} + w_4 \cdot S_{f,shear} \tag{1}$$

$$\text{subject to} \quad g_i \leq 0 \qquad i = 1, 2, 3, \ldots, m \tag{2}$$

where w_i are weight values that account for the ratio between unit costs, $V_{f,flex}$ and $V_{f,shear}$ are, respectively, the volume of composite used for flexural and shear reinforcement and $S_{f,flex}$ and $S_{f,shear}$ estimate the surface of the interface on to which the adhesive must be applied. The design recommendations of [1] are formulated through constraint functions g_i. These parameters depend on the following design variables:

- in the case of the flexural strengthening system [Figure 2], elastic modulus ($E_{f,flex}$) and ultimate strength ($f_{fu,flex}$) of the FRP and width ($b_{f,flex}$), thickness ($t_{f,flex}$) and length ($L_{f,flex}$) of the plate

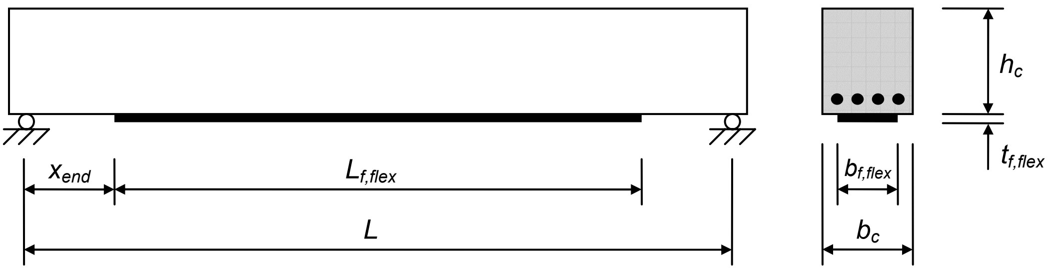

Fig. 2 Flexural strengthening system

- in the case of the shear strengthening system [Figure 3], elastic modulus ($E_{f,shear}$) and ultimate strength ($f_{fu,shear}$) of the FRP, inclination ($\theta_{f,shear}$) of the fibres, width ($b_{f,shear}$), perimeter ($p_{f,shear}$) and thickness ($t_{f,shear}$) of the sheet or strip, number ($n_{f,shear}$) of sheets or strips and separation ($s_{f,shear}$) between them

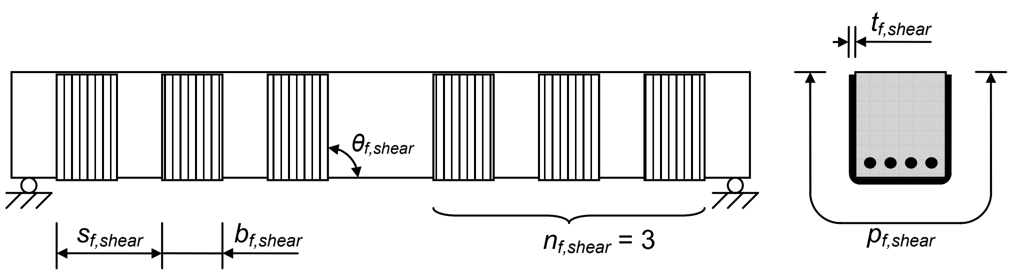

Fig. 3 Shear strengthening system

Some of the variables are of a discrete nature, and the search space of solutions for the optimization problem is controlled by the commercial standards available. Two databases have been prepared with the specifications of a manufacturer of FRP reinforcements for concrete structures. The database on Table 1 is used for flexural strengthening. Two series of CFRP (carbon fibre reinforced polymer) soffit plates are considered, with elastic moduli of 165 GPa or 210 GPa and ultimate strengths of 2800 MPa or 2500 MPa, respectively. The plates have a standard thickness of 1.2 mm, but 2.4 mm thick plates are formed by using two plates. With standard widths of 50 mm or 80 mm a number of combinations are made to produce total widths from 50 mm up to 550 mm.

Table 1 Database for CFRP soffit plate

Number	$E_{f,flex}$ *(GPa)*	$f_{fu,flex}$ *(MPa)*	$b_{f,flex}$ *(mm)*	$t_{f,flex}$ *(mm)*
1	165	2800	50	1.2
2	165	2800	80	1.2
3	165	2800	2x50	1.2
4	165	2800	3x50	1.2
5	165	2800	2x80	1.2
—	—	—	—	—
62	210	2500	6x80	2x1.2
63	210	2500	10x50	2x1.2
64	210	2500	11x50	2x1.2

For shear strengthening [see database on Table 2], only U-jacketed sheets have been considered (see Section 3.2 for details). The depth of the shear CFRP sheets is supposed to fully cover each side of the beam [as represented in Figure 3] and have a standard width $b_{f,shear}$ = 300 mm. Two fibre inclinations have been considered, 90º or 45º. The brochure includes values for separation $s_{f,shear}$ of 300 mm (continuous sheets), 400, 450, 500, 550, 600, 650 and 700 mm. According to the specifications of the manufacturer, once a ply of CFRP has been installed (wet lay-up), its thickness is about 1 mm and its mechanical properties are $E_{f,shear}$ = 63 GPa and $f_{fu,shear}$ = 700 MPa. Up to 4 plies are considered as a possible solution.

Table 2 Database for CFRP U-sheets

Number	$E_{f,shear}$ *(GPa)*	$f_{fu,shear}$ *(MPa)*	$\theta_{f,shear}$	$b_{f,shear}$ *(mm)*	$t_{f,shear}$ *(mm)*	$s_{f,shear}$ *(mm)*
1	63	700	90º	300	1	300
2	63	700	90º	300	1	400
3	63	700	90º	300	1	450
—	—	—	—	—	—	—
62	63	700	45º	300	4x1	600
63	63	700	45º	300	4x1	650
64	63	700	45º	300	4x1	700

3 DESIGN CONSTRAINTS

The design recommendations on flexural and shear strengthening of RC beams proposed by FIB (International Federation for Structural Concrete) [1] in accordance with Eurocode 2 [7] have been considered. These design recommendations are based on limit-states-design principles and constitute a first step for reaching a European standard on externally bonded FRP reinforcements for civil engineering applications. Specific partial safety factors are defined for carbon fibre (CFRP), glass fibre (GFRP) and aramid fibre (AFRP). The limit states for ordinary RC structures are slightly redefined in order to account for the specific failure modes of FRP strengthened members. In this section, a brief description is made of the design limit states considered by the optimization routine, as well as the corresponding constraint function associated to each one of them.

3.1 Flexural strengthening

The required moment strength M_{Sd} of a section is calculated using the load factors defined in [7]. The resisting bending moment M_{Rd} of an FRP strengthened concrete member can be calculated assuming full composite action, strain compatibility, internal force equilibrium and taking into account the failure mode of the repaired beam [see Figure 1]. For the calculation of M_{Rd}, a non-linear analysis has to be performed to obtain the internal strains of the FRP strengthened RC section that satisfy force equilibrium, without developing any failure mode. A parabolic-rectangular stress-strain curve is adopted for concrete in compression, with an ultimate compression strain of ε_{cu} = 0.35%. A bilinear stress-strain is adopted for the internal steel reinforcement. And for the composite material, the stress-strain curve is assumed to be linear until rupture; the FRP design rupture strain is defined as it follows:

$$\varepsilon_{fd,flex} = \frac{f_{fu,flex}}{\gamma_{f,flex}\ E_{f,flex}} \tag{3}$$

where the values $f_{fu,flex}$ and $E_{f,flex}$ are obtained from the manufacturer's specifications and $\gamma_{f,flex}$ is the partial safety factor for the FRP soffit plate, which is given in [1]. A partial safety coefficient ranging from 1.20 to 1.35 for CFRP is suggested. The most usual failure modes are concrete crushing and FRP plate debonding. FRP rupture is rarely reported since on the RC beam tension face, plate debonding strain is generally reached earlier than the FRP design rupture strain $\varepsilon_{fd,flex}$. Moreover, experimental results show that, when CFRP plates are used, failure due to plate debonding usually prevails over concrete crushing. Two of the three approaches described in [1] for the assessment of interfacial debonding at flexural cracks have been considered.

Approach A.1 suggests the use of a FRP tensile strain limitation $\varepsilon_{db,flex}$ ranging from 0.65% to 0.85%, based on the results from the German Institute of Construction. Therefore, the tensile strain $\varepsilon_{f,flex}$ of the FRP must satisfy:

$$\varepsilon_{f,flex} \leq \min\left[\varepsilon_{fd,flex}\ ;\ \varepsilon_{db,flex}\right] \tag{4}$$

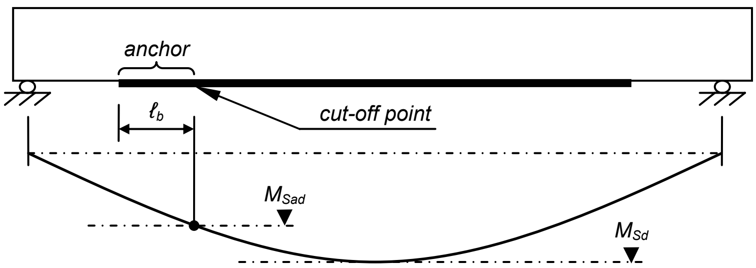

Fig. 4 Anchoring of the FRP soffit plate

Additionally, the model by Neubauer & Rostasy [1, 8] is used to check the adequate anchorage of the plate end in order to avoid plate end interfacial debonding. According to this philosophy, the FRP plate should extend a certain anchoring length ℓ_b beyond a cut-off point [see Figure 4]; the force absorbed by the soffit plate beyond this point (which is the anchorage force) is limited by the maximum force that can be effectively developed through the bond. There is a value of the bond length ($\ell_{b,max}$) such that longer values do not improve the anchorage condition, which means that there is a maximum force $N_{fa,max}$ which cannot be exceeded. For bond lengths ℓ_b shorter than $\ell_{b,max}$, a parabolic law is suggested for estimating the maximum effective anchor force N_{fa} that can be developed through the interface [see Figure 5]. A discussion about the bond strength evaluation can be found in [9].

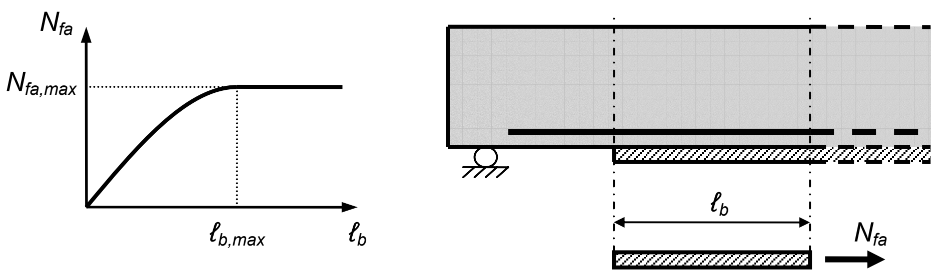

Fig. 5 Anchoring force to bonding length

Approach A.2 is based on the Niedermeier model [1, 10], which is valid both for the assessment of intermediate interfacial debonding and plate end interfacial debonding, as well. This approach takes into account the crack spacing on the concrete tension face and, what is more important, makes the plate be anchored at the uncracked zones of the concrete tension face along the span. That means that the anchoring length ℓ_b is measured from the point at which the applied moment M_{Sad} [represented in Figure 4] equals the cracking moment M_{cr} of the RC beam. As in the previous model, the bond length determines the level of stress that can be absorbed by the FRP without bond failure. A parabolic law, analogous to the one represented in Figure 5 is suggested:

$$\sigma_{fad}\left(\ell_b\right) = \sigma_{fad,max} \frac{\ell_b}{\ell_{b,max}}\left(2 - \frac{\ell_b}{\ell_{b,max}}\right) \tag{5}$$

Expressions for calculating both $\sigma_{fad,max}$ and $\ell_{b,max}$ are given in the FIB document [1]. For plate end interfacial debonding, the FRP stress absorbed at the cut-off point must not exceed σ_{fad} ($\ell_{b,max}$). In the case of intermediate sections, some complex closed form expressions are given in [1], but for this paper a simplification presented by Aprile & Benedetti [11] is used. This model assumes that the available anchorage length $\ell_{b,int}$ for intermediate sections of the plate is the minimum value between $\ell_{b,max}$ and the mean separation ℓ_{cm} between two adjacent cracks:

$$\ell_{b,int} \le \min\left[\ell_{b,max}\ ;\ \ell_{cm}\right] \tag{6}$$

$$\sigma_{f,flex} \le \min\left[\sigma_{f,yield} + \sigma_{fad}\left(\ell_{b,int}\right);\ {f_{fu,flex}}/{\gamma_{f,flex}}\right] \tag{7}$$

Eq. (7) expresses the stress limitation on the FRP for intermediate regions, where:

- $\sigma_{f,yield}$ is the tensile stress in the FRP when the longitudinal steel reinforcement starts yielding
- σ_{fad} ($\ell_{b,int}$) is calculated through Eq. (6) by substituting ℓ_b for the available anchorage length $\ell_{b,int}$

Observe that Eq. (7) is equivalent to Eq. (4) from approach A.1; the strain compatibility of the FRP strengthened section depends on the FRP stress limitation of Eq. (7). For the calculation of the mean separation ℓ_{cm} between adjacent cracks, the formulation reported in the latest version of Eurocode 2 is used, with a slight modification described in [11] to account for the contribution of the FRP plate.

Given that the flexural failure of a FRP strengthened member is more brittle than that of an ordinary RC beam, the FIB document suggests that, for providing adequate ductility, the internal steel should have sufficiently yielded at failure. Accordingly, an upper-limit is defined for the ratio ξ of the neutral axis depth y at ultimate to the effective beam depth d and the following constraint is used. That limit is ξ_{lim} = 0.45 for concrete types C35/45 or lower and ξ_{lim} = 0.35 for higher concrete types.

Shear crack interfacial debonding is a subject which has not been sufficiently developed by the research community yet. Nevertheless, FRP flexural-strengthened RC beams are particularly prone to this type of debonding at regions where the vertical shear force reaches its highest values. The FIB document [1] refers to the models proposed by Blaschko [12] and Matthys [13].

Additionally, concrete cover separation failure [see Figure 1] is one of the most frequently reported. It seems to be typical of short plates, i.e plates that are not anchored at the uncracked zones of the beam span. However, concrete cover rip-off has also been seldom reported in plates ending close to the supports [11]. Several numerical models have been proposed but, generally, poor agreement has been found with experimental data. This failure is started with shear cracks that appear at the plate ends but propagate on the plane of the steel reinforcement and its assessment is usually associated with the shear force. The FIB document refers to the model by Jansze [14], which defines an upper limit $V_{Rd,\ rip\text{-}off}$ for the factored shear force V_{Sd} (x_{end}) acting at the end of the FRP soffit plate. Expressions for the estimation of $V_{Rd,\ rip\text{-}off}$ are given in [1].

The effect of the externally bonded FRP reinforcement on the serviceability can be assessed using the transformed section analysis. To prevent damage or excessive creep of the concrete and steel yielding under service loads, a strengthened member should satisfy the provisions of Eurocode 2. Mainly, limitations on the stresses of the concrete in compression and the steel reinforcement are applied, as well as a specification stated in the FIB document to prevent creep rupture of the FRP:

$$\sigma_{f,flex} \le \eta\ f_{fu,flex} \tag{8}$$

where $\sigma_{f,flex}$ is the maximum tensile stress in the FRP plate under service loads and η is the FRP stress limitation coefficient at service loads. From experimental creep tests, an indicative value of η = 0.8 has been suggested for CFRP.

The GA based optimization routine follows a stochastic search pattern in which plate dimensions are those of Table 1. The plate widths $b_{f,flex}$ range from 50 mm to 550 mm, which means that it is possible to choose a wider FRP soffit plate than the available width b_c on the RC beam tension face. Therefore a geometrical constraint function is added to ensure that $b_{f,flex}$ does not exceed b_c.

3.2 Shear strengthening

The factored shear force V_{Sd} applied on the beam is calculated according to [7]. Thorough research for the evaluation of the resisting shear force V_{Rd} provided by the FRP shear strengthening of RC beams has been relatively limited. Most models opt for simplify the problem to some extent by making an analogy between the FRP reinforcement and the internal steel stirrups. Although the tensile stress absorbed by the FRP fibres is assumed to be uniform, the local stress of a particular fibre depends on several factors, such as its position in relation to the shear crack, the available anchoring lengths above and below the point at which the fibre is intersected by the diagonal crack and, most important, the type of configuration used for the FRP shear strengthening system [see Figure 6].

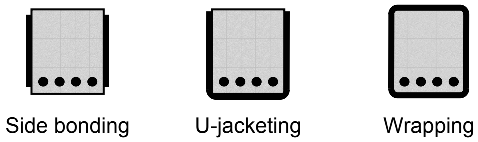

Fig. 6 FRP configurations for shear strengthening

Several studies by Täljsten, Triantafillou and Antonopoulos [15-17] show that when the shear strengthened RC member reaches the shear failure, the tensile strain of the fibres is less than the tensile fracture strain $\varepsilon_{fu,shear}$. At the ultimate state, a certain degree of debonding at the concrete-FRP interface is observed, which is mainly due to excessive straining of the FRP. Thus, an effective strain $\varepsilon_{fe,shear}$ is defined, which depends on the configuration of the FRP reinforcement, the available bond length (similarly to the case of soffit plates), the stiffness of the FRP system and the tensile strength of the concrete. The FIB document treats the externally bonded shear strengthening system as a kind of stirrups. Thus, the shear capacity of the RC beam is:

$$V_{Rd} = \min\left[V_{Rd2}\ ;\ V_{cd} + V_{wd} + V_{fd,shear}\right] \tag{9}$$

Eq. (9) is formulated in accordance with Eurocode 2, where appropriate expressions for V_{Rd2}, V_{cd} and V_{wd} are given. The contribution $V_{fd,shear}$ of the FRP reinforcement is:

$$V_{fd,shear} = 0.9\ \varepsilon_{fed,shear}\ E_{f,shear}\ \rho_{f,shear}\ b_c\ d\ \left[\cot(\theta) + \cot(\theta_{f,shear})\right]\ \sin(\theta_{f,shear}) \tag{10}$$

where $\varepsilon_{fed,shear}$ is the design value of the effective FRP strain, $\rho_{f,shear}$ is the ratio of FRP shear reinforcement and θ is the angle of the diagonal shear crack with respect to the member axis (assumed equal to 45º). As stated above, the effective strain $\varepsilon_{fe,shear}$ depends on the shear strengthening configuration. Side bonding proves the less efficient solution. On the contrary, wrapping is the most efficient one, but its application is difficult if at least one side of the beam is not accessible, as is the case with slab-supporting beams. U-jacketing is moderately effective (tough it may need mechanical anchors for some applications) and is the one that has been used for creating the database of solutions for the optimization routine [see Table 2].

The assessment of the shear capacity of the strengthened beam must be performed at positions located at a distance higher than the effective beam depth d from the supports. Depending on the number of U-jacketed sheets, it is possible that an intermediate region of the span remains non-strengthened for shear. In Figure 7, the position of the third U-sheet from the left in relation to the represented diagonal shear crack is considered the most effective one [5]. Let $x_{u,shear}$ be the distance from the starting point of that crack to the nearest support; for the optimization algorithm presented in this paper, it is assumed that distance $x_{u,shear}$ defines the region in which the contribution of the shear strengthening FRP system can be considered, whereas the rest of the beam retains the original non-strengthened value $V_{Rd,o}$ that can be calculated from Eurocode 2.

U-jacketed and fully wrapped FRP shear reinforcements act as mechanical anchors of the flexural soffit plate [5] so, if a design solution contains simultaneously flexural and shear strengthening systems, the design constraint associated with concrete cover separation is automatically deactivated.

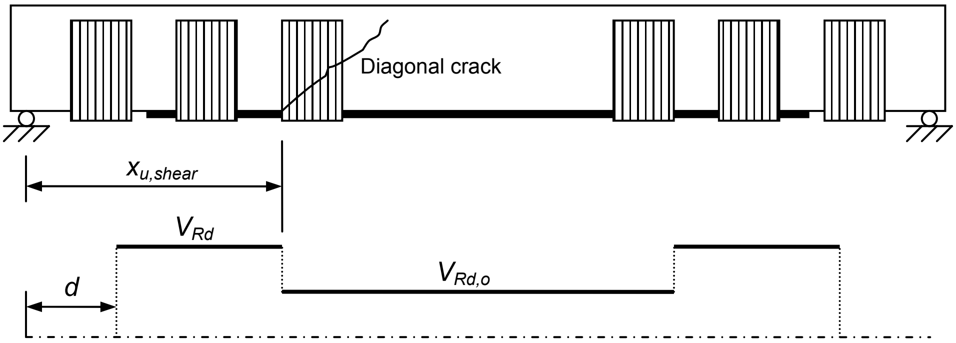

Fig. 7 Shear strengthened and non-strengthened regions

3.3 Constraint handling method

Strengthening designs that do not satisfy the previous requirements must be penalized by an amount depending on the degree of constraint violation. GA based optimization routines are unconstrained optimization techniques; consequently, it is necessary to transform the constrained optimization problem to an unconstrained one. Several methods for handling constraints when GAs are used have been proposed [18]. Among them, the most common approach is the use of a penalty function *P*. The main idea of this method is based on the incorporation of the constraints to the optimization problem through the inclusion of a penalty in the objective function. The value of the penalty is proportional to the degree of violation of a certain solution. Many penalty functions schemes have been proposed for structural design. For this study, a multiplicative form of the objective function proposed by Gen and Chen [19] as a variant of Coit et al's approach [20] has been used.

4 APPLICATION OF THE GENETIC ALGORITHM

GAs are iterative optimum search procedures inspired by the Darwinian principle of evolution. They constitute a stochastic search technique based on the mechanisms of the natural selection and genetics. They were introduced in the 1960s by John Holland [21], developed for engineering applications by Goldberg [22] and are widely used for structural optimization. They are computationally simple, but powerful in their search for improvement and their application does not require derivative information or other auxiliary knowledge. GAs work simultaneously on groups of design points in the whole search space instead of a single design point and can handle continuous, discrete and mixed variable problems.

The GA based optimization routine has been programmed in MATLAB software for the case of a simply supported RC beam. The input data include dimensions of the beam, type of concrete, description of the longitudinal steel reinforcement and of the stirrups, loading condition when the FRP is installed and dead and live loads for the new design. Uniform loads as well as point loads can be considered. Although, the loading scheme can be non-symmetrical, the FRP strengthening scheme is designed symmetrical. The main steps of the algorithm are the following:

a. calculation of the envelopes of the applied bending moment and shear force along the beam, under both ultimate and service conditions
b. generation of a random population of individuals, represented by 22-bit *chromosomes*
c. decoding of each of the chromosomes into candidate design solutions
d. evaluation of the cost *C*, constraints g_i and penalized evaluation function *C'* for each solution
e. calculation of a *fitness* value for each solution, which measures the quality of the solution with respect to the best one within the population
f. application of the *genetic operators*: a *children* population is generated from the current one
g. stop if termination criterion is achieved; otherwise repeat from c. with the children population as new *parents*

Figure 8 shows an example of the chromosome used by the optimization routine. Binary encoding has been used for the present optimization problem:

- the first part consists on a 6 bit string which encodes a number n_1 between 0 and 63; the type of FRP soffit plate for flexural strengthening is defined by n_1 + 1, the result of which is to be looked up in the database of Table 1

0	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0	1	0	0	0	1	1

Soffit plate type $n_1 = 0$ — *Soffit plate end* $n_2 = 13$ — *'U' sheet type* $n_3 = 2$ — *Number of sheets* $n_4 = 3$

Fig. 8 Example of chromosome

- the position $x_{e,flex}$ of the left plate-end [see Figure 9] is calculated through:

$$x_{e,flex} = \frac{n_2 + 1}{128} \cdot L \tag{11}$$

where L is the span length and n_2 is a value between 0 and 63, encoded by a second 6 bit string; notice that if n_2 = 63, the left plate-end would be located at mid-span and this situation would represent the case in which no flexural strengthening device is installed

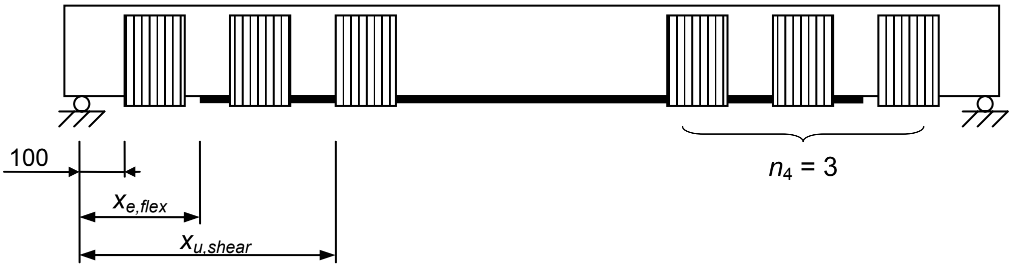

Fig. 9 Example of design solution

- the dimensions and configuration of the U jacketed sheets are encoded by a third 6 bit string that corresponds to a number n_3 between 0 and 63; thus, n_3 + 1 defines the type of FRP shear strengthening system according to Table 2
- the number of U-sheets on each side of the span is n_4, encoded through the last 4 bits

GAs get their power from the genetic operators. The s*election* operator chooses two parents for reproduction; this operation depends on the fitness value of each chromosome. *Crossover* is a reproduction operator, which forms a new chromosome by combining parts of each of the two parental chromosomes. One-point and uniform crossover schemes can be used to perform this operation. *Mutation* forms a slightly modified copy of a selected parent chromosome by altering some of their genes. This operation preserves the diversity among the population and is important for avoiding convergence to local optima. Both mutation and crossover are controlled by probability values. Finally, *pre-selection* ensures that a child will survive into the new generation only if its penalized objective function is better than those of its parents, otherwise the best one of them survives.

5 CFRP PLATE DESIGN OPTIMIZATION EXAMPLE

The following example consists on a four point bending RC beam, with a 4 m span [Figure 10]. The type of concrete is C30/35, the width of the beam is b_c = 300 mm and the total depth is h = 400 mm. The steel reinforcement consists on 2 rebar of ø12 mm plus 2 rebar of ø16 mm, with f_{yk} = 500 MPa. The shear reinforcement consists on ø12 mm stirrups 300 mm. The beam has a flexural capacity $M_{Rd,o}$ = 91.1 m·kN and a shear capacity $V_{Rd,o}$ = 80.5 kN.

The loading of the RC beam consists on a uniform dead load of 15 kN/m plus two point live loads of 55 kN, located at 1000 mm from each support. The maximum factored applied bending moment and shear force are M_{Sd} = 123 m·kN and V_{Sd} = 115.5 kN, respectively (the latter applied at d = 361 mm from the support). Therefore, retrofitting of the beam is required.

A uniform load of 10 kN/m is applied on the beam during the installation of the flexural and shear CFRP reinforcement. The maximum shear force applied within the region located between the point loads is $V_{Sd,central}$ = 40.7 kN, so the RC beam has sufficient shear capacity in the central region. That means that the CFRP shear strengthening must cover a distance $x_{u,shear} \geq 1000$ mm on both sides of the span.

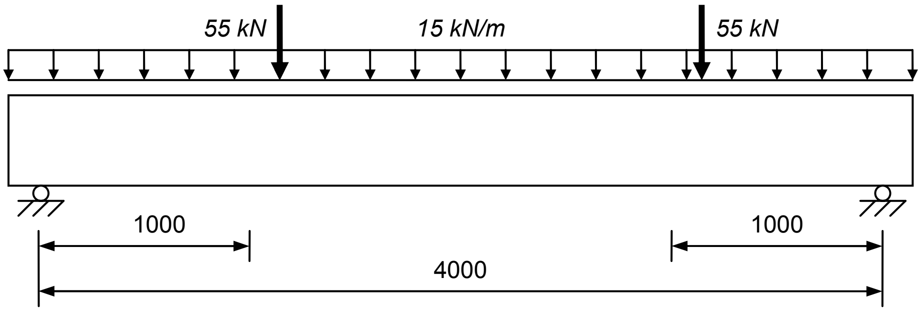

Fig. 10 Application example

The GA based optimization routine presented in Section 4 is run on MATLAB to produce the following results:

Table 3 Optimum shear strengthening scheme for Example 1

GA data: Population size: 400 / Chromosome size: 22 / Uniform crossover / Crossover prob: 85% / Mutation prob: 10%				
CFRP for flexure		CFRP for shear		Constraints
Type	Length	Type	Number	
t_f = 1.2 mm (1 ply) b_f = 3x80 mm E_f = 165 MPa	2813 mm	t_f = 1 mm (1 ply) b_f = 300 mm s_f = 650 mm θ_f = 45º	2 (+2)	None violated

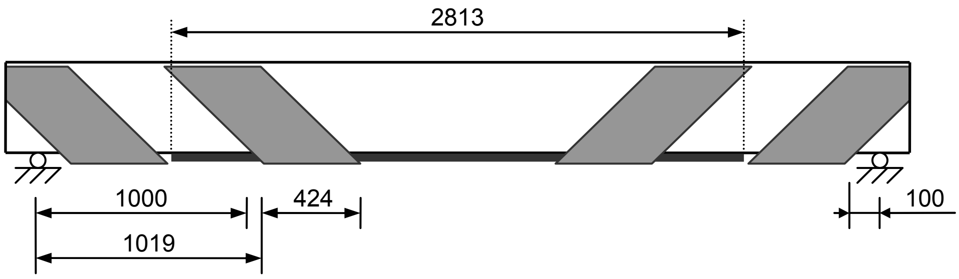

Fig. 11 Solution for Example 1

An off-set distance of 100 mm is defined for every design [see Figure 11] to mark the beginning of the shear strengthening. The configuration described in Table 3 provides a flexural capacity of M_{Rd} = 183 m·kN and a shear capacity of V_{Rd} = 207 kN.

- the design of the soffit plate is not controlled by the maximum applied moment, but by plate end interfacial debonding (approach A.1 has been used); the optimum plate has a bond length $\ell_{b,max}$ = 185 mm that provides an anchoring moment of M_{Rad} = 97 m·kN
- on the other hand, the shear reinforcement seems to be oversized, but actually it is the minimum configuration for covering $x_{u,shear}$; with vertical oriented fibres, a separation higher than 700 mm between U sheets (which is the maximum of the database) would be required, otherwise 3 U sheets would have to be installed on both sides of the span

6 CONCLUSIONS

An optimum design algorithm for flexural and shear strengthening of RC beams strengthened with FRP has been presented. The proposed procedure minimizes the material cost associated to this kind of strengthening while satisfying the serviceability and strength requirements of the code proposal from European Community. A methodology for implementing commercial standard solutions has been described, as well. As research on this reinforcing technique progresses and revisions of the existing guidelines appear, the introduction of new design constraints is simple and automatic since neither explicit variable functions nor gradient information are required by the optimization algorithm itself. Finally, a simple example has given an idea of the complex interaction between the design variables and constraints, yet the algorithm has proved to be simple, systemic and automatic.

REFERENCES

[1] The International Federation for Structural Concrete, Task Group 9.3, *Bulletin 14 – Externally bonded FRP reinforcement for RC structures. Design and use of externally bonded fibre reinforced polymer reinforcement (FRP EBR) for reinforced concrete structures,* Lausanne, Switzerland, 2001.

[2] The Concrete Society, *Technical Report 55 – Design guidance for strengthening concrete structures using fibre composite materials*, Crowthorne, 2000.

[3] The American Concrete Institute, ACI Committee 440, *ACI 440.2R-02 – Guide for the design and construction of externally bonded FRP systems for strengthening concrete structures*, Farmington Hills, Michigan, USA, 2002.

[4] The Canadian Network of Centres of Excellence on Intelligent Sensing for Innovative Structures, ISIS Canada, *Design Manual No.4 – Strengthening reinforced concrete structures with externally-bonded fibre reinforced polymers*, 2001.

[5] Teng, J.G., Chen, J.F., Smith, S.T. and Lam L., *FRP strengthened RC structures*, John Wiley and Sons, 2002.

[6] Oehlers, D.J. and Seracino, R., *Design of FRP and steel plated RC structures: Retrofitting beams and slabs for strength, stiffness and ductility*, Elsevier, 2004.

[7] *EN 1992-1-1:2004 Eurocode 2. Design of concrete structures. Part 1: General rules and rules for buildings*, British Standards Institute, 2004.

[8] Neubauer, U. and Rostasy, F.S., *Design aspects of concrete structures strengthened with externally bonded CFRP-plates*, 7th International Conference on Structural Faults and Repairs, 1997;2, pp. 109-118

[9] Smith, S.T. and Teng, J.G., *FRP-strengthened RC beams. I. Review of debonding strength models*, Engineering Structures, 2002;24, pp. 385-395.

[10] Blaschko, M., Niedermeier, R. and Zilch, K., *Bond failure Modes of Flexural Members Strengthened with FRP*, 2nd International Conference on Composites in Infrastructure, 1998.

[11] Aprile, A. and Benedetti, A., *Coupled flexural-shear design of R/C beams strengthened with FRP*, Composites: Part B, 2004;35, pp. 1-25.

[12] Blaschko, M., *Strengthening with CFRP*, Münchner Massivbau Seminar, TU München (in German), 1997.

[13] Matthys, S., *Structural behaviour and design of concrete members strengthened with externally bonded FRP reinforcement*, PhD dissertation, Ghent University, 2000.

[14] Jansze, W., *Strengthening of reinforced concrete members in bending by externally bonded steel plates*, PhD dissertation, TU Delft, The Netherlands, 1997.

[15] Täljsten, B., *Defining anchor lengths of steel and CFRP plates bonded to concrete*, International Journal of Adhesion and Adhesives, Vol. 17, No.4, pp. 319-327

[16] Triantafillou, T.C., *Shear strengthening of reinforced concrete beams using epoxy-bonded FRP composites*, ACI Structural Journal, Vol. 95, No.2, pp. 107-115

[17] Triantafillou, T.C. and Antonopoulos, C.P., *Design of concrete flexural members strengthened in shear with FRP*, Journal of Composites for Construction ASCE, Vol. 4, No.4, pp.198-205

[18] Coello, C.A., *Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art*, Computational Methods in Applied Mechanics and Engineering, 2002;191, pp. 1245-1287.

[19] Gen, M. and Cheng, R., *A survey of penalty techniques in genetic algorithms*, 6th International Symposium on Robotics and Manufacturing, Montpellier, France, 1996.

[20] Coit, D.W., Smith, A.E. and Tate, D.M., *Adaptive penalty methods for genetic optimization of constrained combinatorial problems*, INFORMS Journal on Computing, 1996;8-2, pp. 173-182.

[21] Holland, J., *Adaptation in natural and artificial systems*, MIT Press, Cambridge, Ma., 1975.

[22] Goldberg, D., *Genetic algorithms in search, optimization and machine learning*, Addison-Wesley, 1989.